

Grade 8 Mathematics, Quarter 2, Unit 2.1  
**Solving Real-World Word Problems  
Involving Volume**

**Overview**

**Number of instructional days:** 5 (1 day = 45–60 minutes)

**Content to be learned**

- Understand the relationship between a cylinder and a cone and use this information to derive the formula for the volume of a cone.
- Understand the relationship between a cylinder and a sphere and use this information to derive the formula for the volume of a sphere.
- Model the volume of a cylinder as a representation of layers.
- Verify the formula for the volume of a cylinder using a net.

**Mathematical practices to be integrated**

- Make sense of problems and persevere in solving them.
- Use concrete objects or pictures to help conceptualize or solve the problem.
- Model with mathematics.
- Apply proportional reasoning to plan or analyze a problem in the community.
- Look for and make use of structure.
- Sort a collection of shapes according to how many sides the shape has.

**Essential questions**

- How does the formula for the volume of a cylinder relate to the number of layers within the three-dimensional figure?
- What is the relationship between the volume of a cone and the volume of a cylinder?
- What is the relationship between the volume of a cylinder and the volume of a sphere?
- If a cone, a cylinder, and a sphere all have the same radius and height, how do their volumes compare?
- If the volume of a prism is determined using a unit cube, and then again with a cube which has been doubled, how would the volumes compare?

## Written Curriculum

### Common Core State Standards for Mathematical Content

#### Geometry

**8.G**

#### Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.

8.G.9 Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.

### Common Core State Standards for Mathematical Practice

#### 1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

#### 4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

#### 7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may

sort a collection of shapes according to how many sides the shapes have. Later, students will see  $7 \times 8$  equals the well remembered  $7 \times 5 + 7 \times 3$ , in preparation for learning about the distributive property. In the expression  $x^2 + 9x + 14$ , older students can see the 14 as  $2 \times 7$  and the 9 as  $2 + 7$ . They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see  $5 - 3(x - y)^2$  as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers  $x$  and  $y$ .

## Clarifying the Standards

### *Prior Learning*

In kindergarten through third grade, students identified and described shapes and their attributes. In sixth grade, students determined the area of right triangles, other triangles, and special quadrilaterals. In seventh grade, students solved real-world problems involving cubes and right prisms. Students also found the volume and surface area of three-dimensional objects composed of cubes and right prisms.

### *Current Learning*

Students derive the volume of cylinders, cones, and spheres. They solve real-world problems using the volume of cylinders, cones, and spheres. Students extend understanding of volume from right prisms to cylinders, and they model the volume of a cylinder as a representation of layers within a three-dimensional figure.

### *Future Learning*

In future courses, students will explain volume formulas and use them to solve problems. They will identify the shapes of two-dimensional cross sections of three-dimensional objects, and they will identify three-dimensional objects generated by rotations of two-dimensional objects. In future courses students will give an informal argument for the formulas of a cylinder, pyramid, and cone.

## Additional Findings

Students may need a review of area prior to working with volume. In addition, students may find it difficult to understand volume as a set of layers when working with non-rectangular prisms.



Grade 8 Mathematics, Quarter 2, Unit 2.2  
**Knowing and Applying Radicals  
and Integer Exponents**

**Overview**

**Number of instructional days:** 15 (1 day = 45–60 minutes)

**Content to be learned**

- Write numbers in scientific notation and in standard form.
- Determine units of appropriate size for very large and very small quantities.
- Apply the properties of integer exponents to generate numerical expressions.

**Mathematical practices to be integrated**

Reason abstractly and quantitatively.

- Create a coherent representation at hand considering the units involved.
- Make sense of quantities and their relationships.

Construct viable arguments and critique the reasoning of others.

- Justify conclusions and communicate them to others.
- Make plausible arguments.
- Analyze situations by breaking them into cases.

Look for and make use of structure.

- Discern a pattern or structure.
- Look for and express regularity in repeated reasoning.
- Look for calculations that are repeated for general methods and for shortcuts.

**Essential questions**

- How do you write a very large or small number in scientific notation?
- How do you write a number in standard form given the number in scientific notation?
- How do you evaluate a number to a negative power?
- How do you express repeated multiplication as an exponential expression?
- How do you multiply and divide exponential expressions with the same base?

## Written Curriculum

### Common Core State Standards for Mathematical Content

#### Expressions and Equations

**8.EE**

#### Work with radicals and integer exponents.

- 8.EE.1 Know and apply the properties of integer exponents to generate equivalent numerical expressions. For example,  $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$ .
- 8.EE.3 Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. *For example, estimate the population of the United States as  $3 \times 10^8$  and the population of the world as  $7 \times 10^9$ , and determine that the world population is more than 20 times larger.*
- 8.EE.4 Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.

### Common Core State Standards for Mathematical Practice

#### 2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

#### 3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an

argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

## 7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see  $7 \times 8$  equals the well remembered  $7 \times 5 + 7 \times 3$ , in preparation for learning about the distributive property. In the expression  $x^2 + 9x + 14$ , older students can see the 14 as  $2 \times 7$  and the 9 as  $2 + 7$ . They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see  $5 - 3(x - y)^2$  as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers  $x$  and  $y$ .

## 8 Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation  $(y - 2)/(x - 1) = 3$ . Noticing the regularity in the way terms cancel when expanding  $(x - 1)(x + 1)$ ,  $(x - 1)(x^2 + x + 1)$ , and  $(x - 1)(x^3 + x^2 + x + 1)$  might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

### Clarifying the Standards

#### *Prior Learning*

In third grade, students learned the relationship between multiplication and division. In fourth grade, students used the factors and multiples of a number. In fifth grade, they interpreted numerical expressions. In sixth and seventh grades, students extended previous multiplication and division rules and extended previous understanding of rational numbers.

#### *Current Learning*

Students apply the property of integer exponents to generate equivalent numerical expressions. They express and interpret numbers in scientific notation.

#### *Future Learning*

In future courses students will apply this knowledge to graphing and writing exponential functions.

### Additional Findings

Students may find it difficult to evaluate negative exponents.





Grade 8 Mathematics, Quarter 2, Unit 2.3

# Graphing and Using Proportional Relationships, Lines, and Linear Equations

## Overview

**Number of instructional days:** 15 (1 day = 45–60 minutes)

### Content to be learned

- Represent a graph or table using  $y = mx + b$ .
- Graph a line given its equation or table.
- Write an equation given a graph or table.
- Interpret distance–time graphs.
- Express slope as a rate of change.
- Interpret the  $y$ -intercept.
- Explain why the slope  $m$  is the same between any two distinct points on a non-vertical line in the coordinate plane using similar triangles.
- Derive the equation  $y = mx + b$  for a line.

### Mathematical practices to be integrated

Model with mathematics.

- Describe how one quantity of interest depends on another.
- Connect the relationships between quantities of interest using such tools as diagrams, two-way tables, graphs, flow charts, and formulas.

Use appropriate tools strategically.

- Use pencil, paper, calculator, protractor, ruler, and any software that might be useful.
- Detect errors by using estimation and other mathematical knowledge.
- Use a mathematical model or technology to make predictions/comparisons of data to deepen students' understanding of concepts.

Look for and make use of structure.

- Look closely to discern a pattern or structure.
- Recognize the significance of a line in a geometric figure for solving problems.

Look for and express regularity in repeated reasoning.

- Utilize a calculator to identify all points on a line.

**Essential questions**

- How do you use equations to represent questions about problem situations and interpret the solutions in the context of the problem?
- How would you graph an equation in  $y = mx + b$  format?
- Given a distance–time graph, how would you interpret what is happening on the graph?
- Given  $y = mx + b$ , can you interpret the meaning of the variables?
- How would you describe the relationship between the variables  $x$  and  $y$  in the equation  $y = 2x + 3$ ?
- Why is the slope  $m$  the same between any two distinct points on a non-vertical line in the coordinate plane?
- How would you derive the equation  $y = mx + b$  for a line?

**Written Curriculum****Common Core State Standards for Mathematical Content****Expressions and Equations****8.EE****Understand the connections between proportional relationships, lines, and linear equations.**

- 8.EE.5 Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.*
- 8.EE.6 Use similar triangles to explain why the slope  $m$  is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation  $y = mx$  for a line through the origin and the equation  $y = mx + b$  for a line intercepting the vertical axis at  $b$ .

**Common Core State Standards for Mathematical Practice****4 Model with mathematics.**

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

## 5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

## 7 Look for and make use of structure.

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### Clarifying the Standards

#### *Prior Learning*

In first through seventh grades, students represented and interpreted data. In fourth and fifth grades, students created equivalent fractions and analyzed patterns and relationships. Sixth-grade students represented and analyzed quantitative relationships between dependent and independent variables. They also used ratio reasoning to solve problems. In seventh grade, students analyzed proportional relationships and used them to solve real-world and mathematical problems.

*Current Learning*

Students understand the connections between proportional relationships, lines, and linear equations. They analyze and solve linear equations and pairs of simultaneous linear equations, and they define, evaluate, and compare functions. In addition, students use functions to model relationships between quantities. Students also use multiple representations for linear functions (tables, graphs, and equations).

*Future Learning*

In future courses, students will create equations that describe numbers or relationships. They will solve systems of equations and will solve equations and inequalities in one variable. Students will also represent and solve equations and inequalities graphically in future courses. They will analyze, create, and build functions using different representations.

**Additional Findings**

Students may struggle with all of the terminology in this standard (dependent and independent variable, vertical line, horizontal line, domain, range, positive and negative slope).